Infrared-finite Observables in N=4 Super Yang-Mills Theory and in N=8 Supergravity

L.Bork[2], D.Kazakov[1,2], G.Vartanov[1,3] and A.Zhiboedov[1,4]

[1]Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna
[2]Alikhanov Institute for Theoretical and Experimental Physics, Moscow, Russia
[3]Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Golm, Germany
[4]Department of Physics, Princeton University, Princeton, NJ, USA

ArXiv: 0908.0387, Phys.Lett. B 681 (2009) 296
ArXiv: 0911.1617, Phys. Rev.D 81 (2010) 105028

Hadron Structure 2010, Gatchina, Russia

Outline

N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons
  All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

N=8 Super Gravity

- $\mathcal{N} = 8$ Super Gravity theory is the most supersymmetric theory possible with gravity and spin ≤ 2
- Field content: 1 massless graviton, 8 massless spin 3/2 gravitinos, 28 spin 1 massless gauge fields, 56 spin 1/2 massless (Majorana) fermions, 70 real massless spin 0 bosons
- The classical theory is conformal invariant, UV divergences cancel on shell up to 7 loops!

N=8 Super Gravity

- $\mathcal{N} = 8$ Super Gravity theory is the most supersymmetric theory possible with gravity and spin ≤ 2
- Field content: 1 massless graviton, 8 massless spin 3/2 gravitinos, 28 spin 1 massless gauge fields, 56 spin 1/2 massless (Majorana) fermions, 70 real massless spin 0 bosons
- The classical theory is conformal invariant, UV divergences cancel on shell up to 7 loops!

N=8 Super Gravity

- $\mathcal{N} = 8$ Super Gravity theory is the most supersymmetric theory possible with gravity and spin ≤ 2
- Field content: 1 massless graviton, 8 massless spin 3/2 gravitinos, 28 spin 1 massless gauge fields, 56 spin 1/2 massless (Majorana) fermions, 70 real massless spin 0 bosons
- The classical theory is conformal invariant, UV divergences cancel on shell up to 7 loops!

AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- What are the quantities that reveal the integrability properties and might be calculated both ways?
- How might PT series be organized to produce simple strong coupling result?

AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- What are the quantities that reveal the integrability properties and might be calculated both ways?
- How might PT series be organized to produce simple strong coupling result?

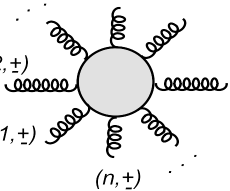
AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- What are the quantities that reveal the integrability properties and might be calculated both ways?
- How might PT series be organized to produce simple strong coupling result?

AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- What are the quantities that reveal the integrability properties and might be calculated both ways?
- How might PT series be organized to produce simple strong coupling result?

Gluon scattering amplitudes



All outgoing gluons with helicity + or -
on mass shell
In the leading N_c order (planar limit)

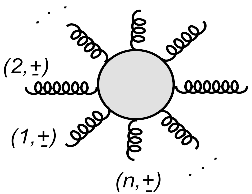
- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \to \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \sum_{perm} Tr(T^{a_{\sigma(1)}}, ..., T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n)}),$$

  where $\mathcal{A}_n$ - physical amplitude, A_n - partial amplitude, a_i - is color index of $i - th$ external "gluon"

- Maximal helicity violating (MHV) amplitudes (two negative helicities and the rest positive) have observed a simple structure on tree level (and even in loops) and one can speculate that this is the consequence of SUSY

Gluon scattering amplitudes



All outgoing gluons with helicity + or -
on mass shell
In the leading N_c order (planar limit)

- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \to \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \sum_{perm} Tr(T^{a_{\sigma(1)}}, ..., T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n)}),$$

where $\mathcal{A}_n$ - physical amplitude, A_n - partial amplitude, a_i - is color index of $i-th$ external "gluon"

- Maximal helicity violating (MHV) amplitudes (two negative helicities and the rest positive) have observed a simple structure on tree level (and even in loops) and one can speculate that this is the consequence of SUSY

Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

-

$$\mathcal{M}_n(\varepsilon) = exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

Cusp anomalous dimension

-

$$F_4^{(1)}(0) = \frac{1}{2}\log^2\left(\frac{-t}{-s}\right) + 4\zeta_2$$

Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

-

$$\mathcal{M}_n(\varepsilon) \quad = \quad exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

Cusp anomalous dimension

-

$$F_4^{(1)}(0) = \frac{1}{2}\log^2\left(\frac{-t}{-s}\right) + 4\zeta_2$$

Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

- $$\mathcal{M}_n(\varepsilon) \quad = \quad exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

Cusp anomalous dimension

- $$F_4^{(1)}(0) = \frac{1}{2}\log^2\left(\frac{-t}{-s}\right) + 4\zeta_2$$

Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

- $$\mathcal{M}_n(\varepsilon) = exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

Cusp anomalous dimension

- $$F_4^{(1)}(0) = \frac{1}{2}\log^2\left(\frac{-t}{-s}\right) + 4\zeta_2$$

Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to <u>three</u> loops for <u>four</u> gluons and up to <u>two</u> loops for <u>five</u> gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
  - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to three loops for four gluons and up to two loops for five gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
  - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to <u>three</u> loops for <u>four</u> gluons and up to <u>two</u> loops for <u>five</u> gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
  - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to <u>three</u> loops for <u>four</u> gluons and up to <u>two</u> loops for <u>five</u> gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
  - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to three loops for four gluons and up to two loops for five gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
  - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

Cancellation of IR divergences

- How and where the IR divergences cancel?
- What is left after cancellation of IR divergences?
- Which quantities (S-matrix elements, x-sections, etc) might have a simple (exact) solution?

Cancellation of IR divergences

- How and where the IR divergences cancel?
- What is left after cancellation of IR divergences?
- Which quantities (S-matrix elements, x-sections, etc) might have a simple (exact) solution?

Cancellation of IR divergences

- How and where the IR divergences cancel?
- What is left after cancellation of IR divergences?
- Which quantities (S-matrix elements, x-sections, etc) might have a simple (exact) solution?

Cancellation of IR divergences

- How and where the IR divergences cancel?
- What is left after cancellation of IR divergences?
- Which quantities (S-matrix elements, x-sections, etc) might have a simple (exact) solution?

From partial amplitudes to cross-sections

To obtain the cross sections from partial amplitudes one has to compute the square of them. In the the planar limit it is just:

$$\Phi_n(p_1^\pm, ..., p_n^\pm) = g^{2n-4}(\frac{g^2 N_c}{16\pi^2})^{2l} \sum_{colors} \mathcal{A}_n^{(l)} \mathcal{A}_n^{(l)*} =$$

$$2g^{2n-4} N_c^{n-2}(N_c^2 - 1)(\frac{g^2 N_c}{16\pi^2})^{2l} \sum_{perm} |A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n-1)}, a_n)|^2$$

Then the cross-section is

$$d\sigma_n(p_{in}) = \Phi_n(p_1^\pm, ..., p_n^\pm) d\phi_k,$$

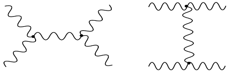
where $d\phi_k$ is the phase space of the outgoing particles:

$$d\phi_k \sim \delta^D(p_{in} - p_{fin}) \mathcal{S}_n \prod_k \delta^+(p_k^2) d^D p_k,$$

$\mathcal{S}_n$ - is the measurement function and integration goes over $D = 4 - 2\varepsilon$ dimensions.

2×2 gluon scattering. Feynman Diagrams

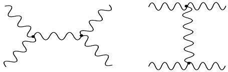
- Tree level



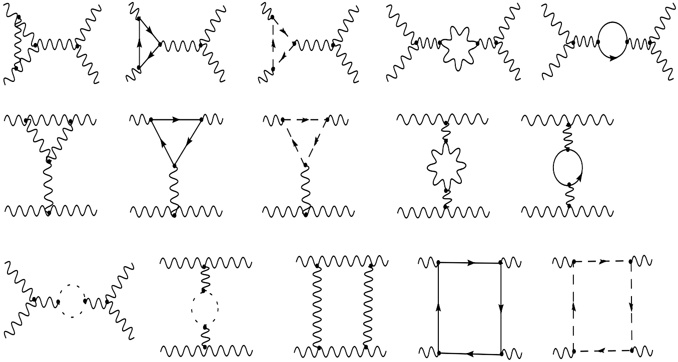
- 1loop

2×2 gluon scattering. Feynman Diagrams

- Tree level



- 1loop



$+$ *permutations*

Virtual Correction (MHV)

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\frac{3+c^2}{(1-c^2)^2}$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})s^2\right.\right.\right.$$
$$\left.+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left.+\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left.+(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

Virtual Correction (MHV)

- Born Term $\quad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\frac{3+c^2}{(1-c^2)^2}$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})s^2\right.\right.\right.$$
$$\left.+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left.+\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$=\frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left.+(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

Virtual Correction (MHV)

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\frac{3+c^2}{(1-c^2)^2}$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})s^2\right.\right.\right.$$
$$\left. +((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left. +\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left. +(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

Virtual Correction (MHV)

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\frac{3+c^2}{(1-c^2)^2}$$

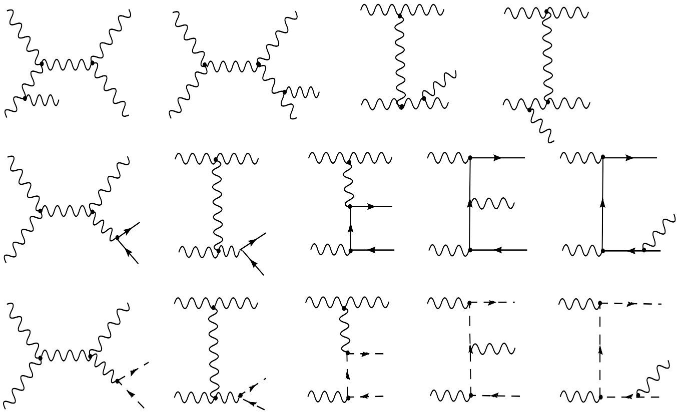
- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})s^2\right.\right.\right.$$
$$\left.+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left.+\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left.+(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

2×3 gluon scattering. Feynman Diagrams

- Tree level



Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2})+\frac{2}{(1-c)^2}\log(\frac{1+c}{2})+\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)}{(1-c^2)^2}\log\frac{(1-\delta)}{\delta}\right]+\text{Finite part}\right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}+\frac{1}{\epsilon}\left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2}+\frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$+\frac{2\delta}{(1-c^2)^2}\left(\frac{2}{3}(5+3c^2)\delta^2-(c^2+19)\delta+2(5c^2+43)\right)+\left(\frac{2(3c^2-24c+85)}{(1-c)(1+c)^3}\log(\frac{1-c}{2})\right.$$
$$-\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log(\frac{1+\delta-(1-\delta)c}{2})-\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.+\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2}-\frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3}+(c\leftrightarrow -c)\right)\right]+\text{Finite part}\right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2}) + \frac{2}{(1-c)^2}\log(\frac{1+c}{2}) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)}{(1-c^2)^2}\log\frac{(1-\delta)}{\delta}\right] + \text{Finite part}\right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon^2}\frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon}\left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2} + \frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$+\frac{2\delta}{(1-c^2)^2}\left(\frac{2}{3}(5+3c^2)\delta^2 - (c^2+19)\delta + 2(5c^2+43)\right) + \left(\frac{2(3c^2-24c+85)}{(1-c)(1+c)^3}\log(\frac{1-c}{2})\right.$$
$$-\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log(\frac{1+\delta-(1-\delta)c}{2}) - \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.+\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} - \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c\leftrightarrow -c)\right)\right] + \text{Finite part}\right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2}) + \frac{2}{(1-c)^2}\log(\frac{1+c}{2}) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)}{(1-c^2)^2}\log\frac{(1-\delta)}{\delta}\right] + \text{Finite part}\right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon^2}\frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon}\left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2} + \frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$+\frac{2\delta}{(1-c^2)^2}\left(\frac{2}{3}(5+3c^2)\delta^2 - (c^2+19)\delta + 2(5c^2+43)\right) + \left(\frac{2(3c^2-24c+85)}{(1-c)(1+c)^3}\log(\frac{1-c}{2})\right.$$
$$-\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log(\frac{1+\delta-(1-\delta)c}{2}) - \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.+\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} - \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c\leftrightarrow -c)\right)\right] + \text{Finite part}\right\}$$

Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2\to 2}^{spl,init} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2;\ i\neq j} d\sigma_{2\to 2}(zp_i, p_j, p_3, p_4)\mathcal{S}_2^{spl,init}(z)$$

- Final splitting

$$d\sigma_{2\to 2}^{spl,fin} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon} d\sigma_{2\to 2}(p_1, p_2, p_3, p_4)\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z)\mathcal{S}_2^{spl,fin}(z)$$

Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider <u>coherent</u> states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2\to 2}^{spl,init} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2;\ i\neq j} d\sigma_{2\to 2}(zp_i, p_j, p_3, p_4)\mathcal{S}_2^{spl,init}(z)$$

- Final splitting

$$d\sigma_{2\to 2}^{spl,fin} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon} d\sigma_{2\to 2}(p_1, p_2, p_3, p_4)\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z)\mathcal{S}_2^{spl,fin}(z)$$

Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2\to 2}^{spl,init} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2;\ i\neq j} d\sigma_{2\to 2}(zp_i, p_j, p_3, p_4)\mathcal{S}_2^{spl,init}(z)$$

- Final splitting

$$d\sigma_{2\to 2}^{spl,fin} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon} d\sigma_{2\to 2}(p_1, p_2, p_3, p_4)\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z)\mathcal{S}_2^{spl,fin}(z)$$

Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon} \sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2\to 2}^{spl,init} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon} \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2;\ i\neq j} d\sigma_{2\to 2}(zp_i, p_j, p_3, p_4) \mathcal{S}_2^{spl,init}(z)$$

- Final splitting

$$d\sigma_{2\to 2}^{spl,fin} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon} d\sigma_{2\to 2}(p_1, p_2, p_3, p_4) \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \mathcal{S}_2^{spl,fin}(z)$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$
$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{-\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}+\text{Finite part}\right\}$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$

$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{-\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}+\text{Finite part}\right\}$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$
$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{-\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}+\text{Finite part}\right\}$$

Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon}\left[\frac{8(c^2+3)}{(1-c^2)^2}\log\delta - \frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$- \frac{4\delta}{(1-c^2)^2}\left(\frac{2}{3}(1+c^2)\delta^2+(c^2-5)\delta+2(c^2+17)\right)+\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.$$
$$+\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-c(1-\delta)}{2}+\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.-\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2}+\frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3}+(c\leftrightarrow -c)\right)\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon}\left[\frac{8(c^2+3)}{(1-c^2)^2}\log\delta - \frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$-\frac{4\delta}{(1-c^2)^2}\left(\frac{2}{3}(1+c^2)\delta^2+(c^2-5)\delta+2(c^2+17)\right)+\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.$$
$$+\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-c(1-\delta)}{2}+\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.-\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2}+\frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3}+(c\leftrightarrow -c)\right)\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon}\left[\frac{8(c^2+3)}{(1-c^2)^2}\log\delta - \frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$-\frac{4\delta}{(1-c^2)^2}\left(\frac{2}{3}(1+c^2)\delta^2+(c^2-5)\delta+2(c^2+17)\right)+\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.$$
$$+\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-c(1-\delta)}{2}+\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.-\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2}+\frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3}+(c\leftrightarrow -c)\right)\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{FnSplit}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{FnSplit}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV}=\frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

- $$B^{AntiMHV}=\frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$

- $$C^{Matter}=\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{Real}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{InSplit}+\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{FnSplit}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{FnSplit}$$

Infrared-free observables

- Registration of two fastest gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3}+B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of one fastest gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3}+B^{AntiMHV}\Big|_{\delta=1/3}+C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1}+C^{Matter}\Big|_{\delta=1}\Rightarrow \text{Finite Part}$$

Infrared-free observables

- Registration of <u>two fastest</u> gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of <u>one fastest</u> gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1/3} + C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \Rightarrow \text{Finite Part}$$

Infrared-free observables

- Registration of two fastest gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of one fastest gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1/3} + C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \Rightarrow \text{Finite Part}$$

Infrared-free observables

- Registration of <u>two fastest</u> gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of <u>one fastest</u> gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1/3} + C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \Rightarrow \text{Finite Part}$$

Infrared-free observables

- Registration of two fastest gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of one fastest gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1/3} + C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \Rightarrow \text{Finite Part}$$

The simplest IR finite answer so far ($Q_f = E$): N=4 SYM Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{AntiMHV} = \frac{\alpha^2 N_c^2}{E^2}\left\{\frac{3+c^2}{(1-c^2)^2}\right.$$
$$-\frac{\alpha N_c}{2\pi}\left[2\frac{(c^4+2c^3+4c^2+6c+19)\log^2(\frac{1-c}{2})}{(1-c)^2(1+c)^4} + 2\frac{(c^4-2c^3+4c^2-6c+19)\log^2(\frac{1+c}{2})}{(1-c)^4(1+c)^2}\right.$$
$$-8\frac{(c^2+1)\log(\frac{1+c}{2})\log(\frac{1-c}{2})}{(1-c^2)^2} + \frac{6\pi^2(3c^2+13)-5(61c^2+99)}{9(1-c^2)^2}$$
$$\left.\left.+2\frac{(11c^3+31c^2-47c+133)\log(\frac{1+c}{2})}{3(1-c)^3(1+c)^2} - 2\frac{(11c^3-31c^2-47c-133)\log(\frac{1-c}{2})}{3(1+c)^3(1-c)^2}\right]\right\}$$

N=8 SUGRA

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{(--++)} = \frac{1}{E^2}\frac{\alpha_{Gr}^2 s^6}{t^2u^2}\left(\frac{\mu^2}{s}\right)^\epsilon = \frac{(\alpha_{Gr}E^2)^2}{E^2}\left(\frac{\mu^2}{s}\right)^\epsilon \frac{16}{(1-c^2)^2},$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{(--++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2})\right.\right.$$
$$\left.\left.+(1-c)\log(\frac{1-c}{2})\right)+2\log(\frac{1+c}{2})\log(\frac{1-c}{2})\right].$$

- Real Emission(MHV)

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)_{Real}^{(--+++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2})\right.\right.$$
$$\left.\left.+(1-c)\log(\frac{1-c}{2})\right)+\text{Finite part}\right].$$

N=8 SUGRA

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{(--++)} = \frac{1}{E^2}\frac{\alpha_{Gr}^2 s^6}{t^2 u^2}\left(\frac{\mu^2}{s}\right)^{\epsilon} = \frac{(\alpha_{Gr}E^2)^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\frac{16}{(1-c^2)^2},$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{(--++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2})\right.\right.$$
$$\left.\left.+(1-c)\log(\frac{1-c}{2})\right)+2\log(\frac{1+c}{2})\log(\frac{1-c}{2})\right].$$

- Real Emission(MHV)

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)_{Real}^{(--+++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2})\right.\right.$$
$$\left.\left.+(1-c)\log(\frac{1-c}{2})\right)+\text{Finite part}\right].$$

N=8 SUGRA

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{(--++)} = \frac{1}{E^2}\frac{\alpha_{Gr}^2 s^6}{t^2u^2}\left(\frac{\mu^2}{s}\right)^{\epsilon} = \frac{(\alpha_{Gr}E^2)^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\frac{16}{(1-c^2)^2},$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{(--++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2}) +(1-c)\log(\frac{1-c}{2})\right) + 2\log(\frac{1+c}{2})\log(\frac{1-c}{2})\right].$$

- Real Emission(MHV)

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)_{Real}^{(--+++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2}) +(1-c)\log(\frac{1-c}{2})\right) + \text{Finite part}\right].$$

N=8 SUGRA

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{(--++)} = \frac{1}{E^2}\frac{\alpha_{Gr}^2 s^6}{t^2u^2}\left(\frac{\mu^2}{s}\right)^\epsilon = \frac{(\alpha_{Gr}E^2)^2}{E^2}\left(\frac{\mu^2}{s}\right)^\epsilon \frac{16}{(1-c^2)^2},$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{(--++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2}) + (1-c)\log(\frac{1-c}{2})\right) + 2\log(\frac{1+c}{2})\log(\frac{1-c}{2})\right].$$

- Real Emission(MHV)

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)_{Real}^{(--+++)} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{64}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left((1+c)\log(\frac{1+c}{2}) + (1-c)\log(\frac{1-c}{2})\right) + \text{Finite part}\right].$$

Initial and Final Splitting

- Initial Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{InSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left(\frac{1-2\delta}{(\delta-1)\delta} - 2\log(1-\delta)\right.\right.$$
$$\left.\left. + 2\log\delta - (1-c)\log\frac{1-c}{2} - (1+c)\log\frac{1+c}{2}\right) + \text{Fimite part}(\delta, c)\right],$$

- Final Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{FnSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\frac{1}{\epsilon}\left[\frac{2\delta-1}{(\delta-1)\delta} + 2\log(1-\delta)\right.$$
$$\left. - 2\log(\delta)\right].$$

Initial and Final Splitting

- Initial Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{InSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left(\frac{1-2\delta}{(\delta-1)\delta} - 2\log(1-\delta)\right.\right.$$
$$\left.\left. +2\log\delta - (1-c)\log\frac{1-c}{2} - (1+c)\log\frac{1+c}{2}\right) + \text{Fimite part}(\delta, c)\right],$$

- Final Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{FnSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\frac{1}{\epsilon}\left[\frac{2\delta-1}{(\delta-1)\delta} + 2\log(1-\delta)\right.$$
$$\left. -2\log(\delta)\right].$$

Initial and Final Splitting

- Initial Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{InSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\left[\frac{1}{\epsilon}\left(\frac{1-2\delta}{(\delta-1)\delta} - 2\log(1-\delta)\right.\right.$$
$$\left.\left. +2\log\delta - (1-c)\log\frac{1-c}{2} - (1+c)\log\frac{1+c}{2}\right) + \text{Fimite part}(\delta,c)\right],$$

- Final Splitting

$$\left(\frac{d\sigma}{d\Omega_{13}}\right)^{(--+++)}_{FnSplit} = \frac{(\alpha_{Gr}E^2)^3}{\pi E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{128}{(1-c^2)^2}\frac{1}{\epsilon}\left[\frac{2\delta-1}{(\delta-1)\delta} + 2\log(1-\delta)\right.$$
$$\left. -2\log(\delta)\right].$$

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty}\int_0^1 dz_1 q_1(z_1,\frac{Q_f^2}{\mu^2})\int_0^1 dz_2 q_2(z_2,\frac{Q_f^2}{\mu^2})\prod_{i=1}^{n}\int_0^1 dx_i q_i(x_i,\frac{Q_f^2}{\mu^2})\times$$
$$\times\, d\sigma^{2\to n}(z_1p_1,z_2p_2,...)S_n(\{z\},\{x\}) = g^4\sum_{L=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
- The dependence on the scale Q_f which characterizes the asymptotic states is left.

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma^{incl}_{obs} = \sum_{n=2}^{\infty}\int_0^1 dz_1 q_1(z_1,\frac{Q_f^2}{\mu^2})\int_0^1 dz_2 q_2(z_2,\frac{Q_f^2}{\mu^2})\prod_{i=1}^{n}\int_0^1 dx_i q_i(x_i,\frac{Q_f^2}{\mu^2}) \times$$
$$\times\, d\sigma^{2\to n}(z_1p_1, z_2p_2, ...)S_n(\{z\},\{x\}) = g^4\sum_{L=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
- The dependence on the scale Q_f which characterizes the asymptotic states is left.

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^{n} \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times$$
$$\times\, d\sigma^{2\to n}(z_1 p_1, z_2 p_2, ...) S_n(\{z\},\{x\}) = g^4 \sum_{L=0}^{\infty} \left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
- The dependence on the scale Q_f which characterizes the asymptotic states is left.

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^{n} \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times$$
$$\times\, d\sigma^{2\to n}(z_1 p_1, z_2 p_2, ...) S_n(\{z\},\{x\}) = g^4 \sum_{L=0}^{\infty} \left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
- The dependence on the scale Q_f which characterizes the asymptotic states is left.

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^{n} \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times$$
$$\times\, d\sigma^{2\to n}(z_1 p_1, z_2 p_2, ...) S_n(\{z\},\{x\}) = g^4 \sum_{L=0}^{\infty} \left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
- The dependence on the scale Q_f which characterizes the asymptotic states is left.

Outlook

- What are the IR safe observables in the strong coupling limit?
- Which IR finite quantities have a simple (integrable) structure?
- What are the true scale invariant quantities in conformal theories?

Outlook

- What are the IR safe observables in the strong coupling limit?
- Which IR finite quantities have a simple (integrable) structure?
- What are the true scale invariant quantities in conformal theories?

Outlook

- What are the IR safe observables in the strong coupling limit?
- Which IR finite quantities have a simple (integrable) structure?
- What are the true scale invariant quantities in conformal theories?

Outlook

- What are the IR safe observables in the strong coupling limit?
- Which IR finite quantities have a simple (integrable) structure?
- What are the true scale invariant quantities in conformal theories?